A Super-Turing (Multi) Membrane Machine for Geometers
(Also for toddlers, and other intelligent animals)
PART 1: Philosophical and biological background
(DRAFT: Liable to change)

PART 2: Towards a specification for mechanisms, is available at
http://www.cs.bham.ac.uk/research/projects/cogaff/misc/super-turing-geom.html

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**Any theory of consciousness that does not include and explain
ancient forms of mathematical consciousness is seriously deficient.**
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Parts of a paper on deforming triangles have been moved into this paper.

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BACKGROUND

Different philosophical and scientific goals

Life is riddled through and through with mathematical structures, mechanisms, competences, and achievements, without which evolution could not have produced the riches it has produced on this planet. That's why I regard evolutionary mechanisms as constituting a Blind Mathematician (as well as being the most creative thing on this planet, as explained in this (draft) paper:
http://www.cs.bham.ac.uk/research/projects/cogaff/misc/creativity.html

Being a "Blind Watchmaker" in Richard Dawkins' sense is a side effect of this.

If AI researchers wish to produce intelligent organisms they will need to understand the deep, pervasive, and multi-faceted roles of mathematics in the production of all organisms, including reproduction and development, in addition to many and varied uses of mathematical mechanisms and competences in meeting the practical information processing challenges of individual organisms interacting with their physical environments and other organisms (including in some cases intelligent prey or predators).

A consequence is that many forms of (human and non-human) consciousness involve deep mathematical (e.g. topological) competences. The ideas of theorists like Immanuel Kant, von Helmholtz, Jean Piaget, Richard Gregory, Noam Chomsky, Max Clowes, Margaret Boden, Daniel Dennett, James Gibson, David Marr and many others (my sample of names should be regarded as quirky and random), all contribute fragments towards a deep theory of consciousness. And all have errors or omissions, either because they focus on a restricted set of phenomena or because their explanatory accounts are inadequate, or both.

For example, any theory of consciousness that says nothing about mathematical consciousness, e.g. the forms of consciousness involved in ancient mathematical discoveries by Archimedes, Euclid, Zeno and others (including pre-verbal human toddlers), must be an incomplete, and usually also incorrect, theory of consciousness. That rules out most of them!

However, "What is it like to be a mathematician?" or "What is it like to understand a mathematical discover?" are not helpful questions about mathematical consciousness. Compare: [What is it like to be a rock?](#) Some of my examples below can be construed as partial answers to "What is it like to be a mathematician?" or "What is it like to make a mathematical discovery?", but the work is still at a stage that's too early for a clear structure to determine the order of presentation.

Types of (meta-) theory about mathematics

In discussing the nature of mathematics and the mechanisms that make mathematical discoveries and their use possible, we need to distinguish the following (the order is temporary, likely to change, and not intended as significant):

- **Biological justification/explanation**
  One way to indicate connections between evolution and mathematics would be to show that evolved control mechanisms have mathematical features (e.g. homeostatic negative feedback loops, or that mechanisms for generating and analysing linguistic syntactic structures in utterances and thoughts use mathematically describable syntactic structures or semantic contents). These mechanisms can be have important biological functions (e.g. use of negative feedback can play a role in maintaining temperature, or concentration of chemicals, or motion towards a target). Many other cognitive functions arguably use mathematical structures and relationships, but not just numerical structures and relationships, e.g. formulation of a goal, comparison of merits of alternative goals, or generation of a plan to build a complex object, or a plan to get to a desired location currently out of sight).

  Biological evolution (The blind mathematician) made many mathematical discoveries and put them to good use in control mechanisms, long before any individual organism was **aware** of using them.

  But what that claim means is not obvious. In part, it involves development and use of powerful "construction kits" with meta-mathematical properties: e.g. the meta-grammatical competences allowing the human genome to produce thousands of languages using grammars with different mathematical properties.

  Some of the mechanisms use abstractions that allow for changing parameters, e.g. a type of organism using an epigenetically modified control mechanism whose parameters change as the size, strength, and speed of the organism change.

  There seems to be a huge variety of such mathematical discoveries, some used in control of physical/chemical growth and development, and others in particular forms of sensing and action control.

  In later stages, evolution provides itself with mechanisms able to discover and use important mathematical structures in forms that can be parametrised so as to produce different examples, either in different species, or in different individuals, or in the same individual at different stages of development.

  The discovery of those "re-usable" and "variable" features can be regarded as meta-mathematical discoveries. They involve generic mechanisms that allow individual organisms to make mathematical discoveries, e.g. about how to derive information about the environment from sensory-motor data, or how to control actions to maintain speed while

avoiding obstacles, and how to manage tradeoffs between speed, accuracy and other features.

Many examples involve using structures and processes in the optic array (*not* just structures in retinal projections) to infer structures of perceived objects and changing relationships between them, including structures and relationships never previously encountered -- a point that is also often made about language understanding: e.g. people reading this sentence for the first time, and constructing an interpretation.

Even if depth measures are not available, inferred 3-D structures can be complex and useful, as shown by work on "scene analysis" in the 1960s and onwards, surveyed in Ballard and Brown (1982). Some of the mechanisms use mathematical aspects of the projection process to derive "reverse" projections that can be useful despite information loss. This generally requires constraint-propagation to remove ambiguities.

Many animals, including pre-verbal human toddlers, can do that sort of thing without knowing that they are doing it or how and why what they do works.

Some competences produced by evolution, or learning, or some combination, involve more complex and messy structures than those normally studied by mathematicians, but that does not make them non-mathematical, e.g. a carnivore purposefully changing topological relationships between parts of a prey animal while dismembering it after capture -- a process that is not normally considered mathematical. If decisions about what to do next are merely innate reflex responses the mathematical "reasoning" must have been done by evolution. But there is too much variation in structures and processes involved in eating (and sharing) prey for every response to have been acquired by evolution. An animal that can work out how to deal with a new configuration, like a crow deciding where to insert the next twig in a part-built nest may be using a mixture of topological and geometric reasoning. Compare Betty the hook-making crow Weir et al. (2002)

Only much later in our evolutionary history could individuals have begun making mathematical discoveries that they were aware of making and using, with the ability to describe them and motivation to try to understand how they worked, and in some cases (much later?) the ability to communicate them to others and debate the merits of alternative modes of reasoning.

Later still, evidence suggests that social/cultural applications, practices, and institutions allowed new forms of discovery and development of mathematical competences and knowledge -- including, in some cases, restricting the processes (keeping knowledge secret).

I suspect there is a vast amount of unrecorded pre-history of human and non-human mathematical competence, that can be inferred only from indirect clues, including varied kinds of proto-mathematical intelligence in non-human species coping with different environments, different physiological needs, and different body structures -- sensor and motor mechanisms. Not all relevant cases are human precursors, or even vertebrates: some of the sensory-motor control mechanisms, e.g. in winged insects catching, escaping, mating, feeding and laying eggs.

Even the amazingly reliable transformations between larval and flying stages via a chemical soup must depend on mathematical properties of the genome and its products, which in turn depend on mathematical features of molecular structures and processes discussed by Schrödinger(1944), and many others influenced by him.

We also need deep new theories about the mechanisms and capabilities of biological evolution, e.g. the (growing) theory of evolved construction-kits of many types.
http://www.cs.bham.ac.uk/research/projects/cogaff/misc/construction-kits.html

- **Philosophical justification**
  Many philosophers have discussed the sense (or senses) in which mathematical claims can be justified, and attempted to justify the methods or criteria used: justifying the justifications. That is not my concern here, though there is a need to explain how the evolved mechanisms work, how they evolved, and what features of the physical universe made them possible.

    Compare: Stewart Shapiro, 2009 We hold these truths to be self-evident: But what do we mean by that? *The Review of Symbolic Logic*, Vol. 2, No. 1
    https://doi.org/10.1017/S175502030909011X

  It may appear that I am using 'self-evidence' as a type of justification. I don't! I am concerned with explanatory mechanisms, not justifications. For a short discussion of 'self-evidence' and how it differs from the notion of non-empirical discovery of necessary truths see:
  http://www.cs.bham.ac.uk/research/projects/cogaff/misc/self-evidence.html
  Extreme (and wrong) answers refer to social conventions, aesthetic/moral decisions, pragmatic claims about usefulness, etc. ....

- **Mathematical arguments supporting claimed discoveries,**
  e.g. by deriving them from other established parts of mathematics, e.g.
    deriving arithmetic from logic;
    deriving theorems about prime numbers from elementary facts about numbers;
    deriving geometry from arithmetic+logic;
    deriving most of Euclidean geometry from some small privileged subset
    deriving Euclidean geometry from point-free topology
        (Whitehead, Dana Scott, etc.)
  others ...

- **Philosophical/scientific explanations of how those discoveries are possible**
  (Immanuel Kant's goal -- and one of mine, since my 1962 DPhil thesis);

- **Scientific explanations of why those mathematical discovery/reasoning mechanisms were selected in biological evolution,**
  E.g. describing the roles that various subsets of the mechanisms play in animal intelligence, including toddler intelligence and deep mathematical discovery processes;

- **Mechanistic explanations of how various processes occur and what they achieve, or fail to achieve.**
  This includes computational/engineering models and theories that explain how certain mechanisms work and prove successful in some tasks but not others and why they are useful for organisms or intelligent machines.
  This requires deep analysis of what exactly needs to be understood and proved, including how the machines work and what alternatives there might be.

- **Philosophical/metaphysical ("grounding") explanations**
  of how mathematical truths and proofs can exist in or be relevant to successful and problem solving in a physical universe, or in this physical universe.
  The "how they can exist" part can be regarded as a claim about "grounding" (a currently fashionable label for an old metaphysical idea).

- **Turing's thoughts about intuition vs ingenuity:**
  I suspect that when Turing wrote (Turing 1939, Sec. 11)

  > "Mathematical reasoning may be regarded rather schematically as the exercise of a combination of two faculties, which we may call intuition and ingenuity. The activity of the intuition consists in making spontaneous judgments which are not the result of conscious trains of reasoning. These judgments are often but by no means invariably correct. . . . The exercise of ingenuity in mathematics consists in aiding the intuition through suitable arrangements of propositions, and perhaps geometrical figures or drawings."

  he was moving toward ideas something like the ideas presented here. But that assumes a connection between his thinking in the late 1930s and his thinking around 1950. For a more detailed summary and discussion of his views see:
  http://www.cs.bham.ac.uk/research/projects/cogaff/misc/turing-intuition-sloman.html also
  http://www.cs.bham.ac.uk/research/projects/cogaff/misc/turing-intuition-sloman.pdf

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EVOLVED MATHEMATICAL COMPETENCES
In the other document

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- Much of Jean Piaget's work is also relevant, especially his last two (closely related) books written with his collaborators:
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  Vol 1. The role of possibility in cognitive development (1981)
  Vol 2. The role of necessity in cognitive development (1983)
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