Route Learning Through Classification

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Abstract. A simple approach to route following is to scan the environment and move in the direction that appears most familiar. In this paper we investigate whether an approach such as this could provide a model of visually guided route learning in ants. As a proxy for familiarity we use the learning algorithm Adaboost [6] with simple Haar-like features to classify views as either part of a learned route or not. We show the feasibility of our approach as a model of ant-like route acquisition by learning a non-trivial route through a real-world environment using a large gantry robot equipped with a panoramic camera.

1 Introduction

Individual ant foragers show remarkable navigational performance, rapidly learning long idiosyncratic routes through cluttered environments [2], guided by learnt visual landmark information [16, 10, 4, 14, 5]. Studies of such visual navigation have revealed how insects combine simple strategies to produce robust behaviour and insect navigation is now an established model system for investigations of the sensory, cognitive and behavioural strategies that enable small-brained animals to learn and utilise complex sequences of behaviour in the real world.

One elegant use of visual landmark information that is part of the insect's navigational toolkit is view-based homing. Behavioural experiments with ants [15, 5] and bees [1] have shown that individuals store 2D retinotopic views of the world as seen from their goal location. Subsequent search for that goal location can be driven by a comparison of their current view of the world and the view stored from the goal location. As this is an efficient and economical way of pin-pointing a location, it isn't a great leap to imagine that knowledge of the world over larger scales, such as routes, could be internalised as a series of stored views that are linked together as a sequence. Route behaviour in this framework would entail homing from one stored view to another. However, recent studies with ants suggest that guidance along routes might not be best served by chains of snapshots. Behavioural experiments suggest that routes can be performed using simpler procedural rules where the rule governing a path segment can be associated with the appropriate visually identified location [3]. Moreover, attempts to model route behaviours using linked view-based homing have shown it to be a non-trivial problem which requires the agent to robustly determine at which point a waypoint should be set during route construction, and deciding when a waypoint has been reached during navigation [11]. Essentially, for robust route navigation, an agent therefore needs place recognition to determine where along the route it is [12]. In conjunction with environmental noise, these problems make robust route navigation a non-trivial aim.

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We begin our study of visually guided routes by drawing a line under previous modelling which defines routes in terms of discrete waypoints. Instead, we define a minimal route learning process in which the route is learnt more holistically. Rather than learning and homing to a set of positions, the agent instead learns a more general mapping which associates the current view *not* with a particular place but instead with a particular action. For an ant or robot that can only translate in one direction relative to its body axis and has a fixed viewing direction, the direction of movement is determined by the viewing direction and visa versa. Thus, if the current retinotopic view is similar to a remembered view, it is likely that the current viewing direction is the also the correct direction to move in. Having constraints on movement direction and viewing direction means that a single view can define not only a location but a direction that should be taken at that place. Crucially however, we do not attempt to learn every view along the route, but instead use them to learn a classifier that can be applied to any view to determine how likely it is that it is part of the route. We suggest this approach is a powerful platform for investigating efficient encoding of route guidance information and how this depends on the visual ecology within which an agent navigates.

This tight coupling of sensation and action allows us to reframe the problem of navigation in terms of a search for the views that are associated with a route. By visually scanning the environment and moving in the direction that is most similar to the views encountered during learning an ant or robot should be able to reliably retrace a given route. Both desert ants and wood ants perform scanning behaviours that would support this approach. When released in an unexpected but familiar place the desert ant melophorus bagoti scan the environment by turning rapidly on the spot. More than one scan maybe performed with short straight runs of a few centimetres separating them before the ant finally sets off in a seemingly purposeful manner. Wood ants exhibit a second form of scanning behaviour. Instead of walking in a straight line, wood ants instead tend to weave a somewhat sinuous path. This has the effect of producing scans of the world centred on the overall direction of movement.

We propose that if ants are able to somehow recognise familiar views, then they can recapitulate routes by simply scanning the environment and moving in the direction that is deemed most similar to the views experienced during learning. Ants could of course simply remember the complete set of views that were experienced during learning, however this approach would result in an extremely high memory load. Instead we propose an approach that involves implicitly modelling the distribution of the views experienced during learning by using a classifier to determine whether a given view comes from part of the learned route or not. Using a classifier provides us with a more compact way of storing the information required to recognise familiar views. Crucially, the approach we employ also provides a measure of the expected uncertainty of the classification (as will be explained later). Here we test the idea of using the familiarity of views as a means of navigation by training a classifier to determine whether a given view is part of the route or not and then using the learned classifier to dictate the direction of movement during route following.

Our results implemented on a real robot indicate that this is indeed a feasible navigational strategy allowing the learning of complex routes through cluttered environments such as might be experienced by a navigating ant.

2 Methods

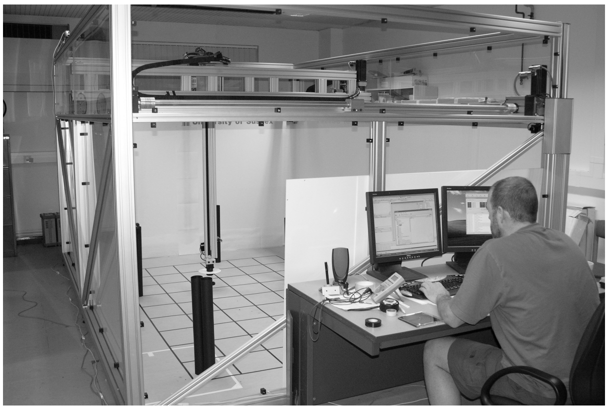

Figure 1. The gantry robot used in all of the experiments.

To test our hypothesis we need to sample the world from an ant's view point. To do this we used a large volume Cartesian XYZ robot to sample panoramic images along a pre-specified ground-level trajectory through a cluttered environment [see Figures 1, 2 and 4].

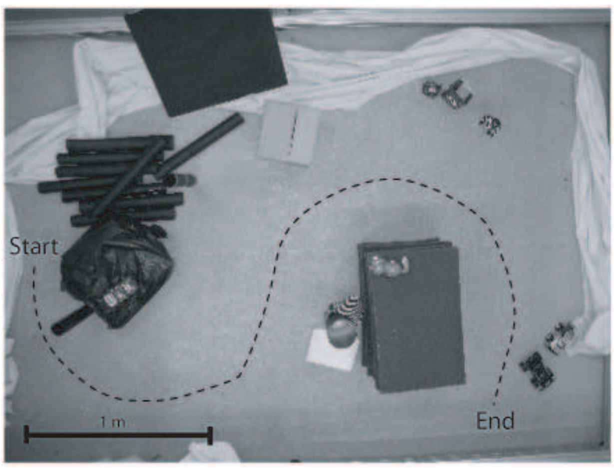

Figure 2. The environment viewed from above showing the learnt route (dotted line) through a cluttered environment. The scale bar indicates 1m.

In order to train a classifier it is necessary to generate positive and negative training examples of the input to be classified. In our case this means collecting views that are part of the route and views that are not part of the route. The positive examples are simply the forward facing views experienced along the route. The negative views consisted of views from the route taken facing to the left and right of the direction of movement at an angle of $\pm 45^\circ$ relative to the route heading. A small amount of normally distributed noise $(s.d. = 0.1 radians)$ was added to each of the sampling directions. This approach is inspired by the observation that ants tend not to move in a straight line on a route but instead proceed in a sinous manner that results in some views that do not relate to the overall direction of travel and some that do [see Figure 3].

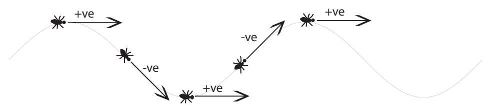

Figure 3. By weaving back and forth during locomotion it is possible to sample both the on-route and off-route views need to train a classifier.

Panoramic images were unwrapped and downsampled before performing feature extraction using simple Haar-like features [Figure 5]. Examples of the unwrapped panoramic images are given in figure 4 which shows an unwrapped image (top) and a heavily downsampled version of the same image (middle) together with the representation of the downsampled image in terms of the activations of a set of 100 Haar-like feature detectors (bottom).

Classifying images is a difficult task due to the high dimensionality of the input if we adopt a pixel by pixel representation. In order to make learning tractable we need to project this high dimensional space into a lower dimensional space that retains enough of the necessary structure to allow successful classification of the input. As a first step in reducing the dimensionality of the input we downsample the images to a resolution of ten degrees per pixel [Figure 4 middle]. To further reduce the dimensionality of the input we use simple Haar-like features [Figure 5] to construct a lower dimensional representation of each image. Each feature produces a single real valued output for a given image so we can can control the dimensionality of the input to our navigational algorithm by defining the number of features that we use. In the current work we chose to use one hundred features selected from a randomly initialised pool of ten thousand. Finally we use the thresholded outputs of the features as simple classifiers. This leaves us with the problem of selecting the features from our pool of ten thousand and determining how to combine their outputs to form a reliable classification. Thankfully, boosting provides us with an approach that achieves both of these requirements.

2.1 Boosting

Boosting is a supervised learning technique for constructing a *strong classifier* from a set of *weak classifiers* given a training set of labelled positive and negative examples. A *weak classifier* is a classifier that performs only slightly better than chance. Conversely, a *strong classifier* is one that performs arbitrarily well. A *strong classifier* is constructed from a linear weighted combination of the outputs of weak classifiers.

There exist many variants of boosting algorithms. Adaboost [6], the approach we use in this paper is one of the most commonly used. The basic algorithm works as follows. At each iteration, the training data are resampled or reweighted according to a distribution of weights that indicate the current importance of each example

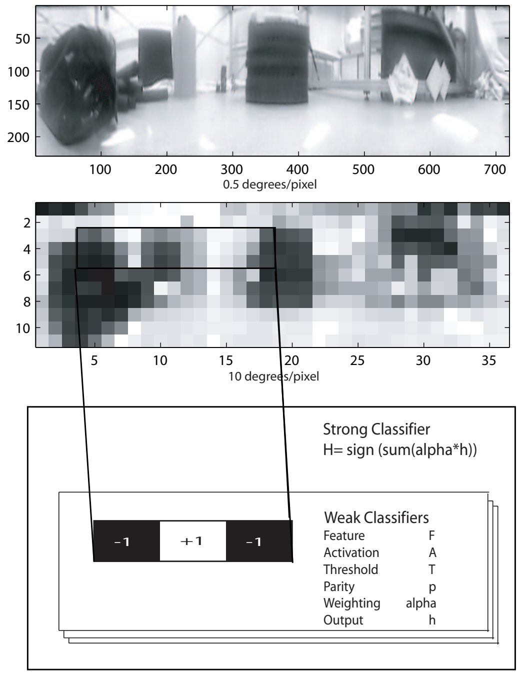

Figure 4. High (top) and low (middle) resolution panoramic images of a typical view from the workspace. The downsampled image is encoded using a set of one hundred Haar-like basis functions (bottom) that are fixed in space relative to the direction of view.

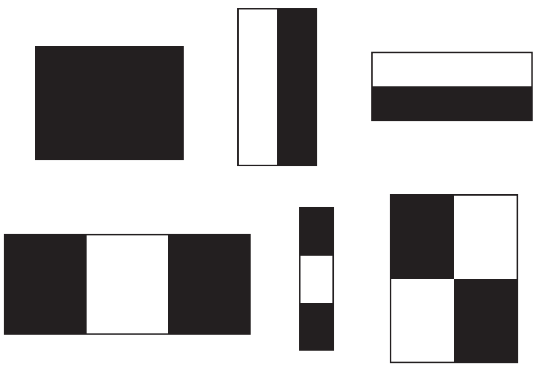

Figure 5. Examples of the six classes of Haar-like features that were used to represent the images. Each feature is defined in terms of a position a size and a class.

in the dataset. A *weak classifier* is then learned using this resampled/reweighted dataset and is added to the *strong classifier*. The relative contribution of each of the *weak classifiers* to the final *strong classifier* is determined by performance on the sampled data. Finally, the weights of incorrectly classified examples are increased and correctly classified examples decreased, thereby encouraging the next *weak classifier* to focus more on the examples that were incorrectly classified at the last iteration. *Weak classifiers* are added until the overall classification performance exceeds some threshold or the maximum number of *weak learners* is reached.

The psuedocode for Adaboost is as follows:

Set T = maximum number of weak classifiers
Given: $(x_1, y_1), \ldots, (x_m, y_m)$
where $x_i \in X,\ y_i \in Y = \{-1, +1\}$
Initialize $W_1(i) = \frac{1}{m}, i = 1, \ldots, m$

For $t = 1, \ldots, T$:
Find the classifier $h_t : X \rightarrow \{-1, +1\}$ that minimizes the error with respect to the distribution W_t

Choose $\alpha_t = \frac{1}{2}\ln\frac{1-\epsilon_t}{\epsilon_t}$ where ϵ_t is the weighted error rate of classifier h_t with respect to the reweighted data.

Update $W_{t+1}(i) = \frac{W_t(i)\exp(-\alpha_t \cdot y_i \cdot h_t(x_i))}{Z_t}$ where Z_t is a normalization factor that ensures that W represents a probability distribution over the training data

Output the final classifier $H(x) = \text{sign}\left(\sum_{t=1}^{T} \alpha_t h_t(x)\right)$

Following Viola and Jones [13] we implement adaboost using single Haar-like features [Figure 5] as the basis of our *weak classifiers*. The Haar-like features consist of randomly chosen rectangular patches of the image which are then subdivided in one of four ways. The value of a *one rectangle feature* is simply the mean intensity value of the patch. The value of a *two rectangle feature* is the difference between the mean intensity of two rectangular regions. The regions have the same size and shape and are located next to each other either horizontally or vertically. The value of a *three rectangle feature* is given by the mean intensities of two outer rectangles subtracted from the mean intensity of a central rectangle, again oriented either horizontally or vertically. Lastly, a *four rectangle feature* computes the difference between diagonal pairs of rectangles. A weak classifier $h_j(x)$ thus consists of a Haar feature f_j, a threshold θ_j and a parity p_j that determines whether the output of the feature detector f_j should be greater than or less than the threshold θ_j.

$$h_j(x) = p_j f_j(x) < p_j \theta_j$$

By providing a pool of feature detectors, each defining a weak learner, adaboost is able to perform feature selection. At each iteration a single feature detector is chosen that best aids in classification. This allows adaboost to pick out and use only those features that are most useful for the current classification problem.

Key to our use of a boosted classifier is the fact that it is possible to obtain a confidence value associated with any given classification made using the trained classifier. This confidence value is related to the margin and is given by:

$$conf = \left\| \sum_{t=1}^{T} \alpha_t h_t(x) \right\|$$

Which is simply the degree to which the the combined weak classifiers differ from zero, prior to the sign being taken.

By applying the classifier to views in different directions we can attempt to determine which of the views are from the learned route. By weighting each of the viewing directions that produce positive classifications by their associated confidence values we can determine a direction to move that is most likely to keep us on the learned route.

2.2 Data collection

All experiments reported here were performed on a gantry robot - a large volume XYZ Cartesian robot. The gantry axis configuration provides an operating volume of 3000 mm X 2000 mm X 2000 mm. The sensor end of the Z-axis can be placed anywhere within this volume with sub-millimetre accuracy. For the experiments presented here a catadioptric camera system (VCAM 360) is mounted on the Z-axis to produce panoramic images. A panoramic mirror projects a 360° image of the environment onto a downward facing CCD video camera. The image is transformed from a circular reflection into a panoramic image that is used for subsequent processing.

The gantry workspace was populated with a variety of objects consisting of foam blocks, piles of fabric, paper rolls and a random selection of toys. Objects were placed in such a way that it was possible to move the sensor head along a route through this visual clutter. Routes could be made more or less challenging by varying the degree of clutter and the straightness of the routes. In order to go beyond what is possible with a snapshot type model, the beginning and end points of all routes were chosen so that it would not be possible to perform the route using this approach. This is achieved by making sure that the end point of the route could not be viewed from the starting position.

2.3 Route learning through classification

Haar-like features were extracted from the set of training images and used to train a boosted classifier. During testing the camera was positioned at the start of the route facing in the correct direction. From this position images were sampled in a range of directions from -60° to $+60^\circ$ in steps of 5° relative to the current heading. Features were extracted from all of these images and used as input to the classifier. All of the viewing directions that produced a positive classification contributed to to a weighted average with the weighting controlled by the confidence interval of the individual classifications. The weighted average was then used to determined the direction of travel and a $5cm$ step was made in this direction. The process was then iterated until success or failure.

3 Results

To test our approach a classifier was trained using a set of images gathered during a single traversal of the route shown in figure 2. The training data consisted of 171 views from 57 positions along the prespecified route, 57 forward facing views and 57 views to both the left and right resulting in 57 positive and 114 negative views. A boosted classifier with 100 weak classifiers was trained on this dataset. Performance was then assessed by starting the robot in a series of different positions close to the original starting position. Figure 6 (top) shows the performance of the approach when starting at the same point as during the training run. Figure 6 (bottom) shows ten separate runs with starting positions at varying distances from the original start point.

The three leftmost starts fail due to the path exiting the permissible workspace of the robot. The fourth leftmost start fails having successfully negotiated the first corner suggesting that the previous three starts would also have failed had they not stopped due to leaving the workspace. The remaining six starts including the original start position (position five) all successfully recapitulate the original route.

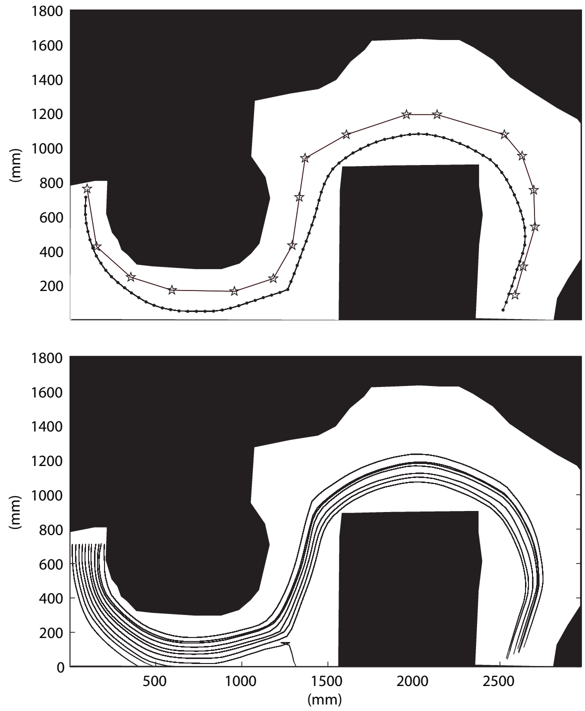

Figure 6. Performance of the algorithm. Top) Training data were collected along the route indicated by the linked stars and the resultant learned route is indicated by the dots. Note the symbols on the two routes do not indicate the points at which images were taken although for the training route they do indicate the waypoints that defined the route. Bottom) Ten example runs of the navigation algorithm. The ten starting positions are centred on the original starting position. Six out of ten trials are successful. Black regions indicate areas of the workspace that the were defined as out of bounds due to potential damage to the camera from collisions with objects.

In order to get an understanding of how the algorithm would perform across the entire environment. We moved the panoramic camera across a grid covering the entire workspace. At each location we scanned in all directions and used the classifier to determine a preferred direction of movement, together with a confidence in its prediction. The results of this analysis are shown in figure 7 with confidence indicated by the length of the arrows. Contrary to what we would expect from the performance observed during route following, when input is sampled from all directions at all positions many views are clearly erroneously classified as being associated with the route as evidenced by the arrows (or lack of them) in Figure 7 that are not consistent with the left to right route on which the classifier was trained.

How can we reconcile this result with the successful performance during our initial experiments? In our route following experiments we did not perform a full 360° scan as we did in constructing the map in Figure 7, but instead limited the scan to forward facing directions. If we construct a map similar to that in Figure 7 but for each point also provide a heading, then we can apply the navigation algorithm exactly as it is used in the first experiment. We determine a heading for each point in space to be the route heading of the nearest section of the route. Figure 8 shows the preferred direction of movement together with confidence values again indicated by the length of the arrows. Note the area where the route failed in the 4th run of the first experiment has zero confidence indicated by the lack of an arrow which explains why the algorithm fails at this point. The algorithm appears to define a corridor along which successful route following is possible with confidence values decreasing the further away from the learned route that the agent strays.

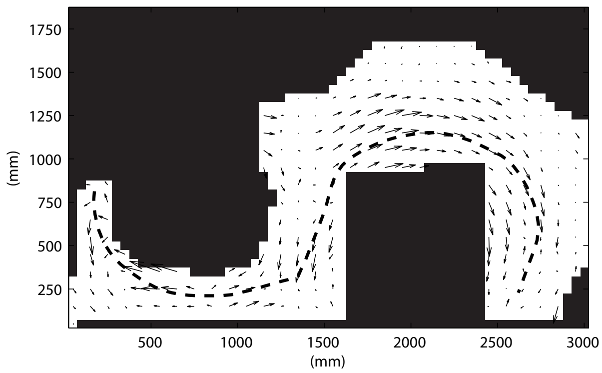

Figure 7. Map of predictions of the route direction based on the route classification algorithm for full 360° scans of the environment sampled across the permissible workspace. Confidence values are represented by the size of the arrows. The dotted line shows the successful route, going from left to right, followed in the first experiment. Black regions indicate areas of the workspace that the were defined as out of bounds due to potential damage to the camera from collisions with objects.

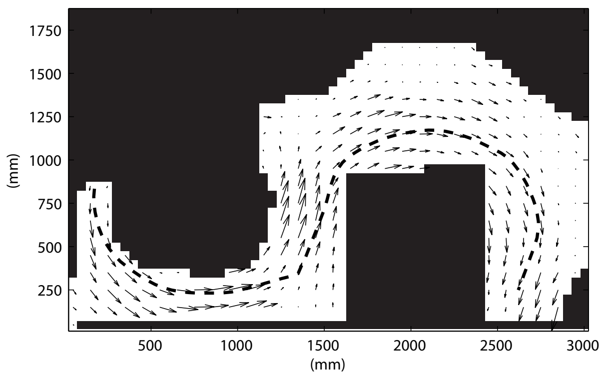

Figure 8. Map of predictions of the route direction based on the route classification algorithm for $\pm 60^\circ$ scans of the environment centred on a heading parallel to the route heading of the nearest section of the original route. Confidence values are represented by the size of the arrows. The dotted line shows the successful route, going from left to right, followed in the first experiment. Black regions indicate areas of the workspace that the were defined as out of bounds due to potential damage to the camera from collisions with objects.

4 Discussion

It is important to note that we are not primarily attempting an engineering solution to the problem of visual navigation. Instead we are trying to gain insight into how ants might learn and use visual information to guide their routes. This is an important point since it means that while successful route following using this approach is a minimum requirement, of potentially greater interest are the limitations and modes of failure that we observe. We have therefore not attempted to improve the overall navigational performance of the approach at the expense of the conceptual simplicity that it represents.

The idea that routes can be learnt using a set of Stimulus-Response (S-R) relations is not new. Equally, it has been observed by various authors that it is possible to orient rotationally by comparing views in different directions to a reference view, effectively resulting in a visual compass [17, 9]. However, combining aspects of these two approaches, as we have done, constitutes a novel approach. Firstly, by parameterising the S-R relationship using a boosted classifier, we not only provide a compact representation of the problem, we also obtain a more robust solution by being less reliant on determining an exact match between the learnt stimulus and the current view. Secondly, by using the classifier to determine view familiarity we are performing recognition rather than recall which is a fundamentally easier problem. In using familiarity rather than similarity to a particular reference view, we can go beyond a simple visual compass and instead construct a method for learning entire routes.

We have shown that it is possible to learn a non-trivial route through an environment using a simple view classification strategy based on positive and negative views collected during a single episode of learning. The route was designed to include high degrees of occlusion and variable depth structure such that a single snapshot taken at the route end-point could not underpin successful navigation through simple image matching. By considering the tight coupling of sensation and action that is present in ants and some robots we were able to reframe the problem of route navigation in terms of a search for simple directional views using a classifier that provides a compact way of storing the information required to recognise familiar views and crucially a measure of the expected uncertainty of the classification.

By embodying the view classifier on a physical platform and constraining the required spatial behaviour to routes, we were able to explore other areas for parsimony. The positive and negative views used by the classifier were collected by simulating a single sinuous path. Consequently, we observed that although spatial knowledge was fragile when the robot was placed at all points in the environment (Figure 7), as long as the agent has some context provided by the likely current direction of travel (Figure 8) the agent can recapitulate a learned route through a visually cluttered world and produce sensible headings from points off the original learned route. This provides an interesting example of where a simple interaction between a behavioural strategy and learnt information provides robust behaviour and without that interaction the agent would require a much more comprehensive survey of the environment. Interestingly, this type of interaction has been observed in ants where directional information from path integration has been shown to increase the precision of visual landmark use [7].

Our ultimate goal with this project is to understand likely and viable mechanisms used by insects for navigation. Therefore it is useful to summarise our framework with respect to some of the desirable properties of insect route behaviour: (i) Route knowledge should be procedural, i.e. an agent should be able to produce the correct behaviour for a given place independently of the prior sequence of visited places. By constraining vision and motion we produced a simple procedural mechanism for visually setting heading which is independent of the sequence of prior visited places. Although as noted above, some degree of hysteresis can be useful to compensate for a sparse set of positive and negative views acquired during training; An interaction which merits further study. (ii) Route knowledge should consist of a broad corridor of familiar places rather than a fragile narrow ridge and agents need to produce sensible behaviour when they are outside the route corridor. In our pilot-study we can satisfy this criteria as the estimates of heading produced from close off-route locations are sensible. Then as one moves further from the route the uncertainty in recalled headings increases which would be a useful signal to commence a systematic search for the route; a behaviour seen in ants when they are lost [8].

Our results suggest that it may be possible for ants and possibly other animals (including humans) to learn routes without the need for recall of the specific views encountered during learning. Instead, recognition together with the simple procedural rule of heading in the direction that appears most familiar may well provide sufficient information to allow successful navigation along routes.

5 Future Work

The ultimate test of our model of route acquisition in ants would require the comparison of the performance our approach with that of an ant in the same environment. There are obvious difficulties in achieving this, however as a first step towards this goal we intend to implement the algorithm on a mobile robot that can then be trained using images collected along an actual foraging route used by ants. It is hoped that by careful manipulation of prominent visual features along the route it will be possible to determine whether or not the approach provides a useful model of real behaviour.

In addition to this there are two main aspects of our approach that we are keen to explore further. These are: (i) How we represent an image. (ii) How we sample the environment to generate positive and negative examples for training a classifier.

In the current set of experiments we chose to represent views using the outputs of a set of Haar-like features. This choice was mainly motivated by the success of this approach when applied to a face detection task [13] where processing speed was a key factor. Since we are less concerned with how fast our code runs we intend to look at the result of employing a more comprehensive feature set consisting of Gabor filters at different positions, scales and orientations. By providing a pool of Gabor filters and learning routes in a variety of different ant-like environments we hope to determine whether there is any consistency in the filters that are selected by Adaboost, i.e. whether there is a general purpose set of filters that will work in a variety of different environments.

Next, we want to look at different ways of generating negative training examples. Potentially any views that are not actually part of the route might be used to define negative instances for the purpose of training. We intend to explore how different, behaviourally plausible, sampling strategies effect the performance of the algorithm. For instance, in the current experiments we only provided negative views to the left and right of the current heading and found that this resulted in classification errors for views from the route facing backwards relative to the learned route direction. By including a wider range of negative views during learning it should be possible to improve the robustness of the approach.

REFERENCES

[1] B. A. Cartwright and T. S. Collett, 'Landmark learning in bees', *Journal of Comparative Physiology A: Neuroethology, Sensory, Neural, and Behavioral Physiology*, **151**(4), 521–543, (1983).

[2] K. Cheng, A. Narendra, S. Sommer, and R. Wehner, 'Traveling in clutter: Navigation in the central australian desert ant melophorus bagoti', *Behavioural Processes*, **80**(3), 261–268, (2009).

[3] M. Collett, T. S. Collett, S. Bisch, and R. Wehner, 'Local and global vectors in desert ant navigation', *Nature*, **394**, 269–272, (1998).

[4] T. S. Collett, E. Dillmann, A. Giger, and R. Wehner, 'Visual landmarks and route following in desert ants', *Journal of Comparative Physiology A: Neuroethology, Sensory, Neural, and Behavioral Physiology*, **170**(4), 435–442, (1992).

[5] V. Durier, P. Graham, and T. S. Collett, 'Snapshot memories and landmark guidance in wood ants', *Current Biology*, **13**(18), 1614–1618, (2003).

[6] Y. Freund and R. E. Schapire, 'A decision-theoretic generalization of on-line learning and an application to boosting', *Computational Learning Theory: Eurocolt 95*, 23–37, (1995).

[7] T. Fukushi and R. Wehner, 'Navigation in wood ants formica japonica: context dependent use of landmarks', *Journal of Experimental Biology*, **207**(19), 3431–3439, (2004).

[8] M. Kohler and R. Wehner, 'Idiosyncratic route-based memories in desert ants, melophorus bagoti: How do they interact with path-integration vectors?', *Neurobiology of Learning and Memory*, **83**(1), 1–12, (2005).

[9] F. Labrosse, 'The visual compass: Performance and limitations of an appearance-based method.', *J. Field Robotics*, **23**(10), 913–941, (2006).

[10] R. Rosengren and W. Fortelius, 'Ortstreue in foraging ants of the formica rufa grouphierarchy of orienting cues and long-term memory', *Insectes Sociaux*, **33**(3), 306–337, (1986).

[11] L. Smith, A. Philippides, P. Graham, B. Baddeley, and P. Husbands, 'Linked local navigation for visual route guidance', *Adaptive Behavior - Animals, Animats, Software Agents, Robots, Adaptive Systems*, **15**(3), 257–271, (2007).

[12] L. Smith, A. Philippides, P. Graham, and P. Husbands, 'Linked local visual navigation and robustness to motor noise and route displacement', in *SAB '08: Proceedings of the 10th international conference on Simulation of Adaptive Behavior*, pp. 179–188, Berlin, Heidelberg, (2008). Springer-Verlag.

[13] P. Viola and M. Jones, 'Rapid object detection using a boosted cascade of simple features', *Computer Vision and Pattern Recognition, IEEE Computer Society Conference on*, **1**, 511–518, (2001).

[14] R. Wehner, 'Visual navigation in insects: coupling of egocentric and geocentric information', *Journal of Experimental Biology*, **199**(1), 129–40, (1996).

[15] R. Wehner, 'The architecture of the desert ant's navigational toolkit (hymenoptera: Formicidae)', *Myrmecological News*, **12**, 85–96, (2009).

[16] R. Wehner and F. Raber, 'Visual spatial memory in desert ants, cataglyphis bicolor (hymenoptera: Formicidae)', *Cellular and Molecular Life Sciences (CMLS)*, **35**(12), 1569–1571, (1979).

[17] J. Zeil, M.I. Hofmann, and J.S. Chahl, 'Catchment areas of panoramic snapshots in outdoor scenes', *Journal of the Optical Society of America A*, **20**(3), 450–469, (2003).